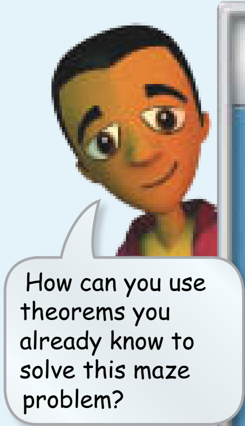
3-3 Proving Lines Parallel

Common Core State Standards

Extends G-CO.C.9 Prove theorems about lines and angles. Theorems include: . . . when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent . . .

MP 1, MP 3, MP 7

Objective To determine whether two lines are parallel

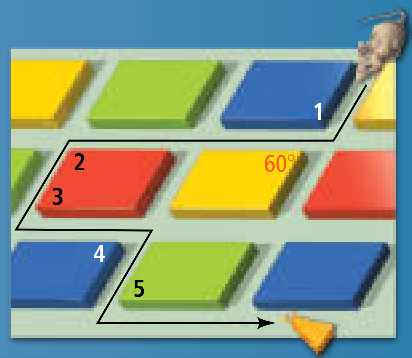


MATHEMATICAL PRACTICES

Lesson Vocabulary
- flow proof

SOLVE IT! Getting Ready!

The maze below has two intersecting sets of parallel paths. A mouse makes five turns in the maze to get to a piece of cheese. Follow the mouse's path through the maze. What are the number of degrees at each turn? Explain how you know.

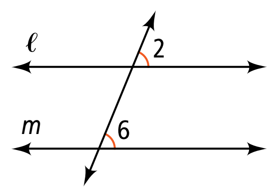


In the Solve It, you used parallel lines to find congruent and supplementary relationships of special angle pairs. In this lesson, you will do the converse. You will use the congruent and supplementary relationships of the special angle pairs to prove lines parallel.

Essential Understanding You can use certain angle pairs to decide whether two lines are parallel.

take note

Theorem 3-4 Converse of the Corresponding Angles Theorem

Theorem	If . . .	Then . . .
If two lines and a transversal form corresponding angles that are congruent, then the lines are parallel.	$\angle 2 \cong \angle 6$ 	$\ell \parallel m$

You will prove Theorem 3-4 in Lesson 5-5.

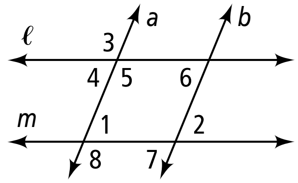
Think

Which line is the transversal for $\angle 1$ and $\angle 2$?
Line *m* is the transversal because it forms one side of both angles.

Problem 1 Identifying Parallel Lines

Which lines are parallel if $\angle 1 \cong \angle 2$? Justify your answer.

$\angle 1$ and $\angle 2$ are corresponding angles. If $\angle 1 \cong \angle 2$, then $a \parallel b$ by the Converse of the Corresponding Angles Theorem.

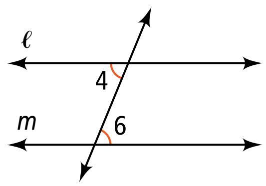


Got It? **1.** Which lines are parallel if $\angle 6 \cong \angle 7$? Justify your answer.

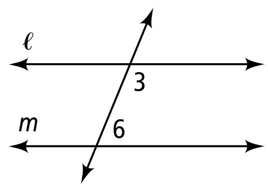
You can use the Converse of the Corresponding Angles Theorem to prove the following converses of the theorems and postulate you learned in Lesson 3-2.

take note

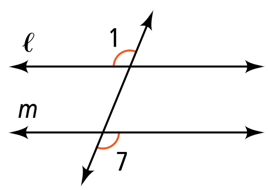
Theorem 3-5 Converse of the Alternate Interior Angles Theorem

Theorem	If . . .	Then . . .
If two lines and a transversal form alternate interior angles that are congruent, then the two lines are parallel.	$\angle 4 \cong \angle 6$ 	$\ell \parallel m$

Theorem 3-6 Converse of the Same-Side Interior Angles Postulate

Theorem	If . . .	Then . . .
If two lines and a transversal form same-side interior angles that are supplementary, then the two lines are parallel.	$m\angle 3 + m\angle 6 = 180$ 	$\ell \parallel m$

Theorem 3-7 Converse of the Alternate Exterior Angles Theorem

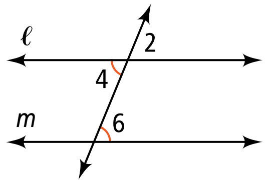
Theorem	If . . .	Then . . .
If two lines and a transversal form alternate exterior angles that are congruent, then the two lines are parallel.	$\angle 1 \cong \angle 7$ 	$\ell \parallel m$

The proof of the Converse of the Alternate Interior Angles Theorem below looks different than any proof you have seen so far in this course. You know two forms of proof—paragraph and two-column. In a third form, called **flow proof**, arrows show the logical connections between the statements. Reasons are written below the statements.

Proof Proof of Theorem 3-5: Converse of the Alternate Interior Angles Theorem

Given: $\angle 4 \cong \angle 6$

Prove: $\ell \parallel m$



$\angle 4 \cong \angle 6$ — Given → $\angle 2 \cong \angle 6$

$\angle 2 \cong \angle 4$ — Vertical ∡ are ≅. → $\angle 2 \cong \angle 6$

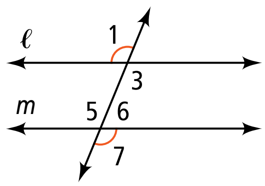
$\angle 2 \cong \angle 6$ — Transitive Property of ≅ → $\ell \parallel m$

$\ell \parallel m$ — If corresp. ∡ are ≅, then the lines are ∥.

Proof Problem 2 Writing a Flow Proof of Theorem 3-7

Given: $\angle 1 \cong \angle 7$

Prove: $\ell \parallel m$



Know
- $\angle 1 \cong \angle 7$

From the diagram you know
- $\angle 1$ and $\angle 3$ are vertical
- $\angle 5$ and $\angle 7$ are vertical
- $\angle 1$ and $\angle 5$ are corresponding
- $\angle 3$ and $\angle 7$ are corresponding

Need

One pair of corresponding angles congruent to prove $\ell \parallel m$

Plan

Use a pair of congruent vertical angles to relate either $\angle 1$ or $\angle 7$ to its corresponding angle.

$\angle 1 \cong \angle 7$ — Given → $\angle 3 \cong \angle 7$

$\angle 3 \cong \angle 1$ — Vertical ∡ are ≅. → $\angle 3 \cong \angle 7$

$\angle 3 \cong \angle 7$ — Transitive Property of ≅ → $\ell \parallel m$

$\ell \parallel m$ — If corresp. ∡ are ≅, then the lines are ∥.

Got It? 2. Use the same diagram from Problem 2 to Prove Theorem 3-6.

Given: $m\angle 3 + m\angle 6 = 180$

Prove: $\ell \parallel m$

The four theorems you have just learned provide you with four ways to determine if two lines are parallel.

Problem 3 Determining Whether Lines are Parallel

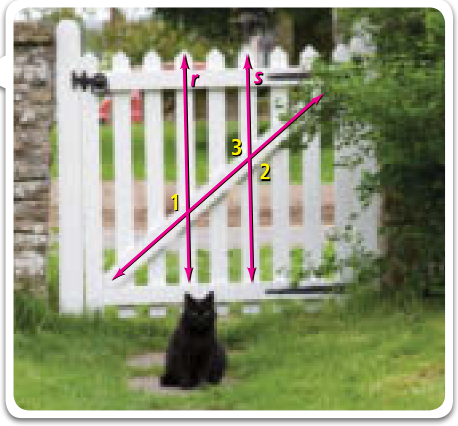
Think

How do ∠1 and ∠2 relate to each other in the diagram?

∠1 and ∠2 are both exterior angles and they lie on opposite sides of the transversal.

The fence gate at the right is made up of pieces of wood arranged in various directions. Suppose ∠1 ≅ ∠2. Are lines *r* and *s* parallel? Explain.

Yes, $r \parallel s$. ∠1 and ∠2 are alternate exterior angles. If two lines and a transversal form congruent alternate exterior angles, then the lines are parallel (Converse of the Alternate Exterior Angles Theorem).



Got It? 3. In Problem 3, what is another way to explain why $r \parallel s$? Justify your answer.

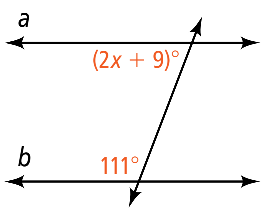
You can use algebra along with the postulates and theorems from Lesson 3-2 and Lesson 3-3 to help you solve problems involving parallel lines.

Problem 4 Using Algebra

Think

Work backward.
Think about what must be true of the given angles for *a* and *b* to be parallel.

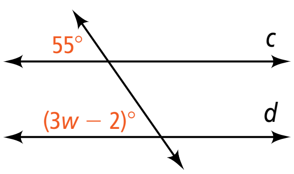
Algebra What is the value of *x* for which $a \parallel b$?



The two angles are same-side interior angles. By the Converse of the Same-Side Interior Angles Postulate, $a \parallel b$ if the angles are supplementary.

$(2x + 9) + 111 = 180$	Def. of supplementary angles
$2x + 120 = 180$	Simplify.
$2x = 60$	Subtract 120 from each side.
$x = 30$	Divide each side by 2.

Got It? 4. What is the value of *w* for which $c \parallel d$?

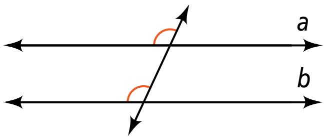


Lesson Check

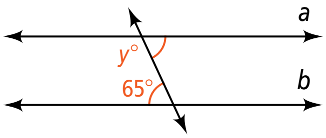
Do you know HOW?

State the theorem or postulate that proves $a \parallel b$.

1.



2.



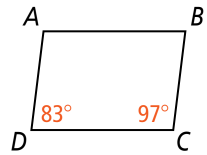
3. What is the value of y for which $a \parallel b$ in Exercise 2?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. Explain how you know when to use the Alternate Interior Angles Theorem and when to use the Converse of the Alternate Interior Angles Theorem.

5. Compare and Contrast How are flow proofs and two-column proofs alike? How are they different?

6. Error Analysis A classmate says that $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$ based on the diagram at the right. Explain your classmate's error.



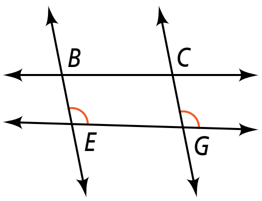
Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Which lines or segments are parallel? Justify your answer.

See Problem 1.

7.



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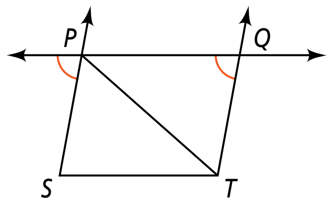

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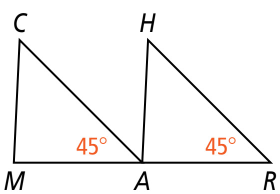

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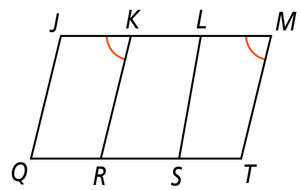
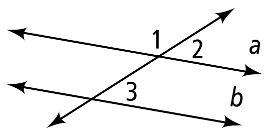


11. Developing Proof Complete the flow proof below.

See Problem 2.

Given: $\angle 1$ and $\angle 3$ are supplementary.

Prove: $a \parallel b$



$\angle 1$ and $\angle 3$ are supplementary.
a. ?

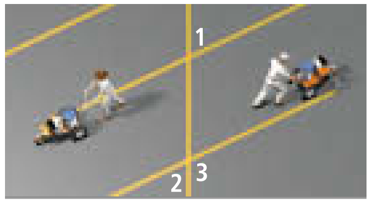
→ **d.** ?
Supplements of the same $\angle$ are $\cong$.

→ $a \parallel b$
e. ?

b. ?
Def. of linear pair

→ $\angle 1$ and $\angle 2$ are supplementary.
c. ?

12. Parking Two workers paint lines for angled parking spaces. One worker paints a line so that $m\angle 1 = 65$. The other worker paints a line so that $m\angle 2 = 65$. Are their lines parallel? Explain.

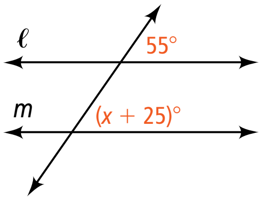


See Problem 3.

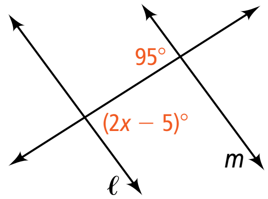
Algebra **Find the value of x for which $\ell \parallel m$.**

See Problem 4.

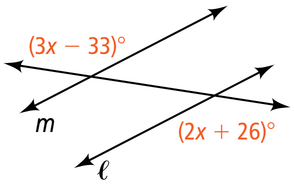
13.



14.



15.



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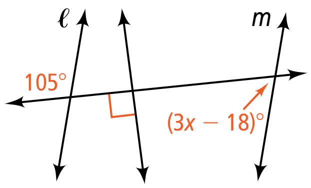
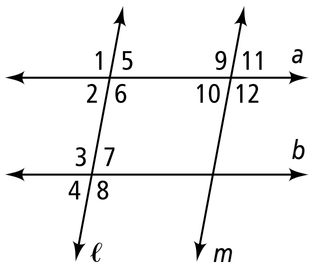


B Apply

Developing Proof **Use the given information to determine which lines, if any, are parallel. Justify each conclusion with a theorem or postulate.**



17. $\angle 2$ is supplementary to $\angle 3$.

18. $\angle 1 \cong \angle 3$

19. $\angle 6$ is supplementary to $\angle 7$.

20. $\angle 9 \cong \angle 12$

21. $m\angle 7 = 65$, $m\angle 9 = 115$

22. $\angle 2 \cong \angle 10$

23. $\angle 1 \cong \angle 8$

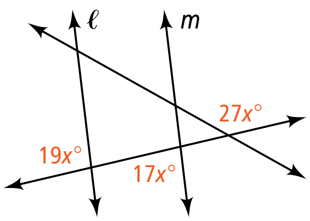
24. $\angle 8 \cong \angle 6$

25. $\angle 11 \cong \angle 7$

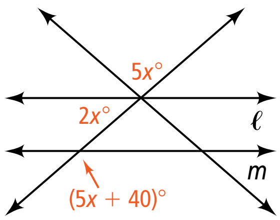
26. $\angle 5 \cong \angle 10$

Algebra **Find the value of x for which $\ell \parallel m$.**

27.



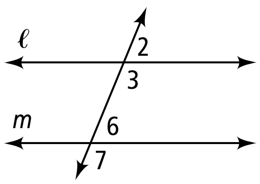
28.



29. Write a paragraph proof.
Proof

Given: $\angle 2$ is supplementary to $\angle 7$.

Prove: $\ell \parallel m$



30. Think About a Plan If the rowing crew at the right strokes in unison, the oars sweep out angles of equal measure. Explain why the oars on each side of the shell stay parallel.

- What type of information do you need to prove lines parallel?
- How do the positions of the angles of equal measure relate?

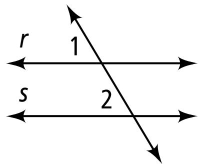
Algebra Determine the value of x for which $r \parallel s$. Then find $m\angle 1$ and $m\angle 2$.

31. $m\angle 1 = 80 - x$, $m\angle 2 = 90 - 2x$

32. $m\angle 1 = 60 - 2x$, $m\angle 2 = 70 - 4x$

33. $m\angle 1 = 40 - 4x$, $m\angle 2 = 50 - 8x$

34. $m\angle 1 = 20 - 8x$, $m\angle 2 = 30 - 16x$



Use the diagram at the right below for Exercises 35–41.

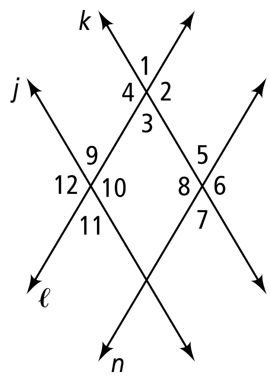
Open-Ended Use the given information. State another fact about one of the given angles that will guarantee two lines are parallel. Tell which lines will be parallel and why.

35. $\angle 1 \cong \angle 3$

36. $m\angle 8 = 110$, $m\angle 9 = 70$

37. $\angle 5 \cong \angle 11$

38. $\angle 11$ and $\angle 12$ are supplementary.



39. Reasoning If $\angle 1 \cong \angle 7$, what theorem or postulate can you use to show that $\ell \parallel n$?

Write a flow proof.

40. Proof **Given:** $\ell \parallel n$, $\angle 12 \cong \angle 8$
Prove: $j \parallel k$

41. Proof **Given:** $j \parallel k$, $m\angle 8 + m\angle 9 = 180$
Prove: $\ell \parallel n$

Challenge

Which sides of quadrilateral *PLAN* must be parallel? Explain.

42. $m\angle P = 72$, $m\angle L = 108$, $m\angle A = 72$, $m\angle N = 108$

43. $m\angle P = 59$, $m\angle L = 37$, $m\angle A = 143$, $m\angle N = 121$

44. $m\angle P = 67$, $m\angle L = 120$, $m\angle A = 73$, $m\angle N = 100$

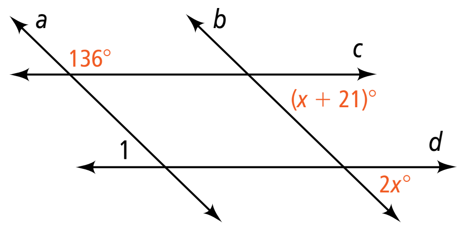
45. $m\angle P = 56$, $m\angle L = 124$, $m\angle A = 124$, $m\angle N = 56$

46. Proof Write a two-column proof to prove the following: If a transversal intersects two parallel lines, then the bisectors of two corresponding angles are parallel. (*Hint:* Start by drawing and marking a diagram.)

Standardized Test Prep

SAT/ACT

Use the diagram for Exercises 47 and 48.



47. For what value of x is $c \parallel d$?

(A) 21 (C) 43

(B) 23 (D) 53

48. If $c \parallel d$, what is $m\angle 1$?

(F) 24 (H) 136

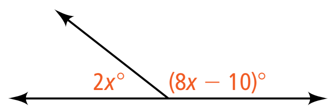
(G) 44 (I) 146

49. Which of the following is always a valid conclusion for the hypothesis?
If two angles are congruent, then __?__.

(A) they are right angles (C) they have the same measure

(B) they share a vertex (D) they are acute angles

50. What is the value of x in the diagram at the right?



(F) $1.\overline{6}$ (H) 17

(G) 10 (I) 19

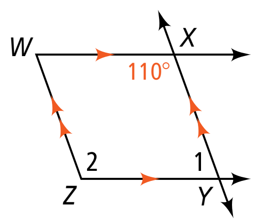
Short Response

51. Draw a pentagon. Is your pentagon convex or concave? Explain.

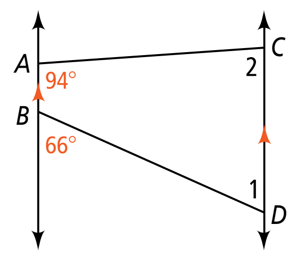
Mixed Review

Find $m\angle 1$ and $m\angle 2$. Justify each answer. See Lesson 3-2.

52.



53.



Get Ready! **To prepare for Lesson 3-4, do Exercises 54–57.**

Determine whether each statement is *always, sometimes,* or *never* true. See Lessons 1-6 and 3-1.

54. Perpendicular lines meet at right angles.

55. Two lines in intersecting planes are perpendicular.

56. Two lines in the same plane are parallel.

57. Two lines in parallel planes are perpendicular.